

Tribhuvan University
Institute of Science and Technology
2065

Bachelor Level/ First Year/ Second Semester/ Science

Computer Science and Information Technology (MTH 155)

(Linear Algebra)

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Candidates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions:

Group A

(10x2=20)

1. Illustrate by example that a system of linear equations has either exactly one solution or infinitely many solutions.
2. What is a linear transformation invertible?
3. Solve the system.
 $3x_1 + 4x_2 = 3$
 $5x_1 + 6x_2 = 7$
by using the inverse of the matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$.
4. State the numerical importance of determinant calculation by row operation.
5. State Cramer's Rule for an invertible $n \times n$ matrix A and vector $b \in \mathbf{R}^n$ to solve the system $Ax = b$. Is this method efficient from computational point of view?
6. Determine if $\{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$, where $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
7. Determine if $W = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is a $Nul(A)$ for $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.
8. Show that 7 is an eigenvalue of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.
9. If $S = \{u_1, \dots, u_p\}$ is an orthogonal set of nonzero vectors in $\mathbf{R}^2$, then S is linearly independent and hence is a basis for the subspace spanned by S .
10. Let $W = \text{span}\{x_1, x_2\}$ where $x_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$. Then construct orthogonal basis for W .

Group B

(5x4=20)

11. Determine if the given set is linearly dependent:

$$(a) \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$$

$$(b) \begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$$

12. Find the 3x3 matrix that corresponds to the composite transformation of a scaling by 0.3, a rotation of 90° , and finally a translation that adds $(-0.5, 2)$ to each point of a figure.

OR

Describe the Leontief Input-Output model for certain economy and derive formula for $(I-C)^{-1}$, where the symbols have their usual meanings.

13. Find the coordinate vector $[x]_B$ of x relative to the given basis $B=\{b_1, b_2\}$, where

$$b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \quad x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

14. Let $A = \begin{bmatrix} 4 & -9 \\ 4 & 8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and basis $B = \{b_1, b_2\}$. Find the B-matrix for the transformation $x \rightarrow Ax$ with $P = \{b_1, b_2\}$.

15. Let u and v be nonzero vectors in $\mathbb{R}^3$ and the angle between them be ϕ . Then prove that

$$u \cdot v = \|u\| \|v\| \cos \phi,$$

where the symbols have their usual meanings.

Group C

(5x8=40)

16. Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Then T is one to one if and only if the equation $T(x) = 0$ has only the trivial solution, prove the statement.

OR

$$\text{Let } A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, \quad u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

and define $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ by $T(x) = Ax$. Then

(a) Find $T(u)$.

(b) Find an $x \in \mathbb{R}^2$ whose image under T is b .

(c) Is there more than one x whose image under T is b .

(d) Determine if c is the range of T .

17. Compute the multiplication of partitioned matrices for

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ 0 & 4 & -2 & 7 & -1 \end{array} \right] \text{ and } B = \begin{bmatrix} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ -1 & 3 \\ 5 & 2 \end{bmatrix}$$

18. What do you mean by change of basis in $\mathbf{R}^n$? Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for $\mathbf{R}^2$ given by $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$.

(a) Find the change of coordinate matrix from C to B .

(b) Find the change of coordinate matrix from B to C .

OR

Define vector space, subspace, basis of a vector space with suitable examples. What do you mean by linearly independent set and linearly dependent set of vectors?

19. Diagonalize the matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$, if possible.

20. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points $(2, 1)$, $(5, 2)$, $(7, 3)$, $(8, 3)$. What do you mean by least squares line?